MAGNITUDE SCALING FACTORS FOR ANALYSIS OF LIQUEFACTION

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ABSTRACT

The "simplified procedure" developed by Seed and Idriss is widely used in the United States to evaluate liquefaction hazard. Empirical evidence suggests that magnitude scaling factors (MSF) required in this procedure are very conservative for moderate-sized earthquakes. We compiled soil and site data for several earthquakes and localities where surface effects of liquefaction did or did not occur. We statistically analyzed these data using logistic regression to develop MSF that have about the same conservatism as other factors in the simplified procedure. Regressed MSF values for magnitude 5.5, 6, and 6.75 earthquakes are 4.5, 2.8, and 1.6, respectively, compared to values of 1.43, 1.32 and 1.13, respectively, as listed by Seed and Idriss. Use of the regressed MSF values may safely reduce calculated liquefaction hazard for moderate-sized earthquakes.

INTRODUCTION

Liquefaction is a phenomenon in which soil, usually sands or silty sands, are transformed from solid state into a liquid state due to shaking from an earthquake. Liquefaction is a major cause of damage during earthquakes and is a necessary consideration in seismic hazard investigations for buildings, pipelines, land use planning, railway and highway bridges, etc. The primary method for analyzing liquefaction hazard used in the United States and many other countries is the simplified procedure developed by Seed and Idriss (1971).

In order to evaluate the liquefaction hazard at a site, the simplified procedure requires several correction factors to account for conditions that differ from those directly analyzed in deriving the technique. The magnitude scaling factor (MSF) corrects the analysis for earthquake magnitudes other than 7.5; the standard used at the time of derivation.

The reliability of magnitude scaling factors developed by Seed and Idriss (1982) has never been confirmed, nor has a statistical analysis been made of the degree of uncertainty or conservatism in the numbers. Empirical evidence suggests that the MSF derived by Seed and Idriss are very conservative, that is, they appear to greatly overestimate the liquefaction hazard for small to moderate size earthquakes (M = 5 to 7).

The purpose of this study is to analyze statistically the reliability of the MSF proposed by Seed and Idriss and to develop a more appropriate set of factors if warranted. Case histories of field performance data were collected for a variety of earthquake magnitudes and soil conditions. Liquefaction hazard was calculated using the Seed and Idriss (1971; 1982) simplified procedure and, these predictions were compared against field performance data. From a statistical analysis of the results, we evaluated the reliability of the Seed and Idriss magnitude scaling factors and develop a statistically derived set of MSF values.

COLLECTION AND CATEGORIZATION OF THE DATA

In order to test the validity of the magnitude scaling factors proposed by Seed and Idriss (1982), we applied the simplified procedure to 412 sites from 42 earthquakes where surface effects of liquefaction were or were not observed. To assure that these sites were accurately analyzed, we re-evaluated many variables which affect soil liquefaction and the best estimates of soil properties and seismic factors were determined for use in the site analyses. A liquefaction analysis program developed at Brigham Young University that applies the Seed and Idriss simplified procedure to borehole logs was used to calculate a prediction of liquefaction occurrence or nonoccurrence. The results from the predicted field performance were then compared with the observed performance.

In order to apply, the Seed and Idriss simplified procedure a compilation of case history data is required. The case history database in this study was developed from several earthquake catalogs: Seed and others (1985), Liao (1986), Ambraseys (1988), and Bartlett and Youd (1992). Case history sites unavailable in these earthquake logs were also added to the database (Loertscher, 1994). Original sources which provided site borehole logs were examined where available to verify the data in the earthquake catalogs.

Goodness-of-fit Statistics

Goodness-of-fit statistics are used in a logistic regression analysis to determine whether the proposed model is statistically significant. The Percent Correctly Predicted (PCP) was the primary method of goodness-of-fit used in this study to determine the adequacy of the logistic regression model. The response for the ith data point is correctly predicted if $Y_i = 1$ and $p_i > 0.5$ or if $Y_i = 0$ and $p_i < 0.5$. The probability, p_i, is calculated after equation (2) has been evaluated for the independent variables and their parameters. Other goodness-of-fit statistics could also be used to determine statistical significance.

LOGISTIC REGRESSION OF LIQUEFACTION DATABASE

With the database of case histories compiled for this study, we performed a regression analysis using the logistic method. The purpose of this logistic analysis to obtain a statistical analysis of the reliability of the MSF proposed by Seed and Idriss. In order to do this a set of scaling factors derived from the logistic analysis will be compared with those published by Seed and Idriss (1982).

The dependent variable, Y_i, used in this study is liquefaction occurrence or nonoccurrence denoted by a 1 or 0 respectively. Independent variables are physical properties of the soil or the earthquake which could affect the occurrence or nonoccurrence of liquefaction. The independent variables used in this study are those used in the Seed and Idriss simplified procedure because these variables are so widely accepted by the geotechnical profession. In the Seed and Idriss simplified procedure, corrected blowcount, $(N_1)_{60}$, and cyclic stress ratio, CSR, are specifically used to determine if liquefaction will or will not occur. In order to calculate a magnitude scaling factor from the logistic analysis, the independent variable of M_w must be used in addition to the other two independent variables.

Because a restricted number of independent variables, $(N_1)_{60}$ and CSR, are used in the Seed and Idriss simplified procedure, different combinations of these variables were used to increase the initial set of candidate variables in the logistic regression, for example, ln(CSR), $M_w(N_1)_{60}$, M_wCSR, etc. Only a limited number of variables and combinations can be made available in the logistic analysis. It is possible that variables other than those used in the Seed and Idriss procedure could be significant but, were not considered in this study.

A logistic regression analysis was performed on the liquefaction database developed for this study using the stepwise regression method. This procedure includes the most significant of the candidate independent variables in the regression model and provides an optimum model for the given set of variables. In the logistic regression analysis on this study's database, the stepwise regression procedure selected the following model:

$$\text{logit}(p) = -13.42 + 3.04M_w - 0.035M_w(N_1)_{60} + 0.41M_w\ln(CSR) \qquad (3)$$

where

logit(p)	=	the log-odds transform, equation (1)
p	=	the probability of liquefaction occurrence

M_w	=	the earthquake moment magnitude
$(N_1)_{60}$	=	the corrected SPT blowcount
CSR	=	the cyclic stress ratio.

Equation (3) allows prediction of the probability of liquefaction occurrence or nonoccurrence due to shaking from a specific earthquake based on the properties of a given site. The derivation of magnitude scaling factors from this logistic regression model is given in a later section.

The Percent Correctly Predicted (PCP) for the resultant model given in equation (3) indicates whether the model is statistically significant. For sites where liquefaction occurred ($Y_i=1$), those with a $p_i > 0.5$ are considered correctly predicted by the model and comprise approximately 92% of the results. For sites where liquefaction did not occur ($Y_i=0$), those with a $p_i < 0.5$ are considered correctly by the model and comprise approximately 85% of the results. The PCP clearly shows that the logistic regression model [equation (3)] correctly predicts a high percentage of the actual empirical data. Other goodness-of-fit statistics also give high values for the logistic regression model.

Probability in the Regression Model

In Seed and Idriss' plot of CSR versus $(N_1)_{60}$, a critical boundary separates liquefaction and no liquefaction sites (see Figure 1). In the simplified procedure for liquefaction hazard analysis, this boundary curve is used to calculate the cyclic stress ratio required to induce liquefaction. Because the Seed and Idriss bounding curve was fit by hand and not as a result of any mathematical or statistical procedure, the curve does not have any unique associated probability of liquefaction occurrence. Due to this fact, the actual probability that liquefaction will occur is unknown for cases when the Seed and Idriss simplified procedure predicts occurrence.

This study uses the concept of having a probability associated with prediction of liquefaction occurrence or nonoccurrence. The logistic regression model [equation (3)] relates the probability of liquefaction occurrence or nonoccurrence to magnitude, blowcount, and cyclic stress ratio. Several curves with different probabilities of liquefaction occurrence can be developed for a specified earthquake magnitude. Figure 2 shows an example of these contours of equal probabilities for magnitude 7.5. If the $(N_1)_{60}$ and CSR for a site give a point on one of these curves, the probability that liquefaction occurs ($Y_i=1$) for that site equals the probability associated with the curve.

Because probability is used in the regression model, several different estimates of scaling factor can be derived for each magnitude. The actual magnitude scaling factor to be used depends upon the degree of certainty desired in the prediction of liquefaction occurrence or nonoccurrence. If a high probability (e.g. p = 95%) is used in a prediction, the user can be 95% certain that a prediction of liquefaction occurrence ($Y_i=1$) is correct. Sites where marginal liquefaction occurs will not be predicted using a magnitude scaling factor with a high probability value. Therefore, the probability of liquefaction occurrence associated with a magnitude scaling factor should be determined according to the degree of conservatism

Values for site variables (see Table 1) used to determine the occurrence or nonoccurrence of liquefaction were compiled for each site. To develop the greatest possible confidence in the predicted results, the values for each variable were examined and evaluated to give the best possible estimate.

LOGISTIC REGRESSION ANALYSIS

The method of logistic analysis is a type of statistical regression analysis. As with all regression analyses, logistic analysis assumes that changes in independent variables, X_i, create corresponding changes in a dependent variable, Y_i. Data with a binary dependent variable use the logistic method for analysis because typical linear or nonlinear regression analyses do not give a statistically sound answer for a noncontinuous variable.

Generalized Procedure

Logistic regression analysis on binary variables is performed in a manner similar to multiple regression with two main exceptions: (a) the response variable is the logit term derived from the dependent variable, Y_i, using equation (1), and (b) parameters, ß's, are interpreted according to effects on the logit term rather than on the dependent variable (Freeman, 1987).

For a given dependent variable, in this case liquefaction occurrence or nonoccurrence, p is the probability that liquefaction will occur ($Y_i=1$) for a set of independent variables, X_i. The probability of no liquefaction occurrence ($Y_i=0$) given the same set of independent variables is 1-p. To use a logistic regression analysis, Y_i must be transformed into a logit term, logit(p), that will be used as the new response variable. The logit term is given by the odds of liquefaction to no liquefaction:

$$\text{logit}(p) = \ln[(p)/(1-p)] \qquad (1)$$

With logit(p) as the response variable, a regression line is fit to the data set with the form given in equation (2).

$$\text{logit}(p) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n \qquad (2)$$

The terms of β_i are parameters determined by the regression analysis. The X_i terms are the independent variables used to determine the probability of the liquefaction occurrence. These independent variables are properties used in the analysis which may or may not affect the dependent variable, for example, physical parameters of the soil or of the earthquake.

The independent variables are entered into the regression model using a stepwise procedure. Given an initial set of candidate independent variables, a stepwise regression analysis performs an automated search for the optimal combination of independent variables, that is, the combination which best explains changes in the dependent variable.

desired.

Another consideration also affects the determination of the probability used in deriving the MSF. If the MSF is to be used to scale the original Seed and Idriss M=7.5 curve, the probability associated with that original curve would be appropriate for use in the derivation of the scaling factors. Unfortunately, the Seed and Idriss bounding curve does not have a unique associated probability.

Thus, there are two factors to consider in determining an appropriate probability for the calculation of the magnitude scaling factors. (a) Use of a probability with enough conservatism to enable sites of marginal liquefaction to be predicted. (b) A probability which is somewhat consistent with the original Seed and Idriss curve for M = 7.5. Comparisons were made between the Seed and Idriss curve and curves with different probabilities of liquefaction occurrence developed from the logistic regression model. The p=32% logistic model curve has some consistency with the mid to upper portion of the Seed and Idriss curve and is also conservative. Points at this curve have a 32% probability that liquefaction will occur ($Y_i=1$). The probability, p=32%, meets the criteria we established and will be used in the derivation of magnitude scaling factors.

MAGNITUDE SCALING FACTORS

To simplify the logistic regression model, equation (3), the logit term is evaluated for p=32%. This simplification leads to an equation for cyclic stress ratio, CSR, in terms of magnitude, M_w, and corrected SPT blowcount, $(N_1)_{60}$.

$$CSR = \exp[-7.400 + 30.826/M_w + 0.0851(N_1)_{60}] \qquad (4)$$

Magnitude scaling factor is defined as the ratio of the CSR for a given earthquake magnitude and the CSR for M_w=7.5 (Seed and Idriss, 1982), as illustrated by equation (5).

$$MSF = (CSR_M \;/\; CSR_{7.5}) \qquad (5)$$

By evaluating equation (4) for M_w = M and M_w = 7.5, respectively, and substituting into equation (5), an equation for magnitude scaling factor is generated. The final equation for MSF at a probability, p=32%, is as follows:

$$MSF = \exp\{30.826[(1/M_w) - (1/7.5)]\} \qquad (6)$$

A comparison of the magnitude scaling factors from the logistic analysis for p=32% and those evaluated for two other probabilities with MSF published in the Seed and Idriss procedure (1982) is given in Table 2.

Recommended uses of the MSF

Magnitude scaling factors developed from the logistic analysis are used in the simplified procedure for liquefaction hazard analysis in the same manner as those developed by Seed and Idriss. Analyses were performed to determine if the use of logistic MSF gives correct prediction of liquefaction occurrence or nonoccurrence for the case history sites in this study's database. The percent of site responses correctly predicted was determined for both the logistic MSF calculated in this study and the MSF published by Seed and Idriss (1982). For all magnitudes, the logistic MSF in this study correctly predicted 91% of the site responses and, the MSF proposed by Seed and Idriss correctly predicted 78% of the site responses. For magnitudes less than 7.5, the logistic MSF in this study correctly predicted 92% of the site responses and, the MSF proposed by Seed and Idriss correctly predicted 63% of the site responses. The MSF proposed by Seed and Idriss gave poor predictions for sites which did not liquefy. The results indicated:

(1) Use of the original Seed and Idriss M=7.5 curve as seen in Figure 1, with the logistic MSF gives good results.

(2) Use of a logistic curve calculated from equation (3) using the appropriate magnitude and probability, p= 32%, also gives good results.

(3) Use of the original MSF with the Seed and Idriss M=7.5 curve is very conservative.

Figure 3 shows an example of these three results. The case history data for magnitude 6.4 earthquakes show sites of liquefaction or no liquefaction. Also shown are three curves. (1) One of these curves is the original Seed and Idriss bounding curve scaled by the logistic MSF generated in this study. This curve gives good results for separating the liquefaction sites from no liquefaction sites. (2) The next curve is a logistic curve calculated from equation (3) for M_w = 6.4 and p = 32%. This curve also gives good results with one nonliquefied site falling on the liquefaction occurrence boundary. (3) The final curve is the original Seed and Idriss bounding curve scaled by the MSF proposed by Seed and Idriss (1982). This curve is conservative and includes four nonliquefied sites within the liquefaction occurrence boundary.

The good fit of the logistic regression model on the empirical case history data suggests that a new set of magnitude scaling factors are warranted. The scaling factors derived from this model predict liquefaction occurrence or nonoccurrence well and should be considered in future analyses.

CONCLUSIONS

The simplified procedure developed by Seed and Idriss (1971; 1982) is the most commonly used method to evaluate a site for liquefaction hazard as a result of earthquake ground motions. The simplified procedure requires several correction factors to account for conditions different from those directly analyzed in deriving the technique. One of these correction factors, the magnitude scaling factor (MSF) adjusts the analysis for earthquake magnitudes other than 7.5, the base value used by Seed and Idriss. In this study, we developed an extensive database from several earthquake catalogs of sites where surface effects of liquefaction did or did not occur. This database was used to analyze statistically the reliability of MSF values published by Seed and Idriss (1982). Regression analyses were performed on this database to determine if a re-evaluation of the Seed and Idriss magnitude scaling factors was warranted. Logistic regression analysis proved to be the most appropriate analysis for the binary data in the case history database.

The variables used in the simplified procedure include cyclic stress ratio, CSR, and corrected SPT blowcount, $(N_1)_{60}$. These same variables were also used in this study to develop magnitude scaling factors that are compatible with the Seed and Idriss procedure. These variables proved to be significant in the final logistic regression model. From that model, we generated a set of magnitude scaling factors (Table 2). Because the analyses were performed on case history data, rather than laboratory test data, as was used by Seed and Idriss, the predicted results more closely match the field occurrence or nonoccurrence of liquefaction.

The goodness-of-fit for the logistic regression model on the empirical case history database suggests that use of this new set of magnitude scaling factors is warranted. The magnitude scaling factors generated from the logistic regression analysis (Table 3) predict liquefaction occurrence and nonoccurrence well and are recommended for use with the Seed and Idriss simplified procedure for liquefaction hazard analysis.

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TABLE 1. Site Condition Variables

Y_L	Liquefaction occurrence (yes=1/no=0)
M	Earthquake magnitude; in this study moment magnitude, M_w
a_{max}	Peak horizontal ground acceleration (g)
σ_o	Total overburden pressure (kg/cm^2)
σ'_o	Effective overburden pressure (kg/cm^2)
r_d	Stress reduction coefficient
$(N_1)_{60}$	Standard penetration test blowcount
FC	Fines content (<0.075 mm) (%)
D_{50}	Median grain size (mm)

TABLE 2. Comparison of Magnitude Scaling Factors from Logistic Analysis and Seed and Idriss (1982)

Magnitude	MSF p=32%	MSF p=50%	MSF p=68%	Seed & Idriss
8 50	0.62	0.60	0.58	0 89
8 00	0.77	0 76	0.75	0 94
7.50	1.00	1.00	1.00	1 00
7 00	1 34	1.36	1.39	1.08
6.75	1.58	1.62	1.67	1.13
6.50	1 88	1 95	2.03	1.19
6 00	2 79	2.97	3.16	1.32
5.50	4.46	4.87	5.32	1.43

TABLE 3. Recommended Magnitude Scaling Factors for use with the Seed and Idriss Simplified Procedure

Magnitude	MSF
8.50	0.62
8.00	0.77
7.50	1.00
7.00	1.34
6.75	1.58
6.50	1.88
6.00	2.79
5.50	4.46

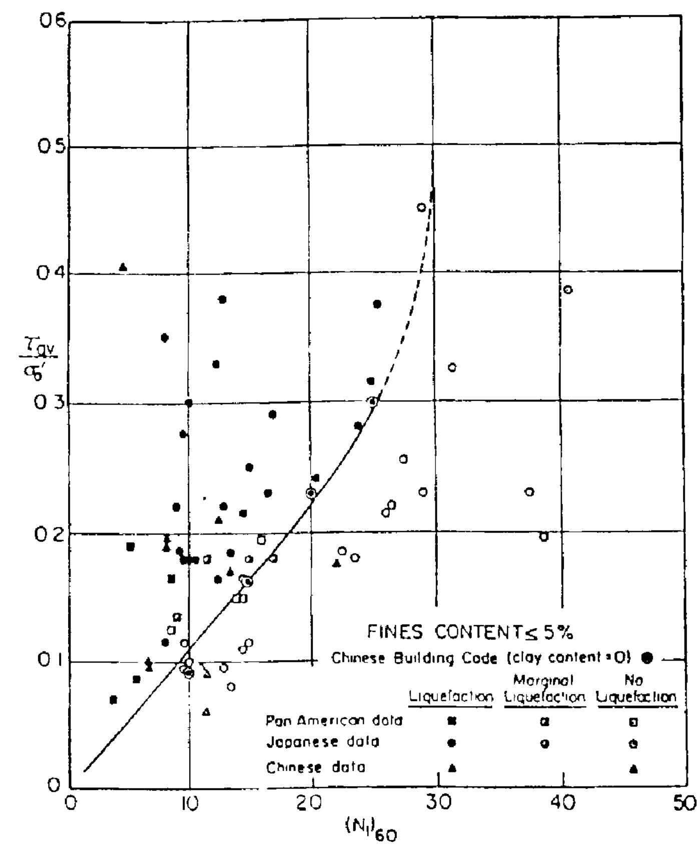

FIGURE 1. Relationship between cyclic stress ratio and $(N_1)_{60}$ for a magnitude 7.5 earthquake (Seed and others, 1985)

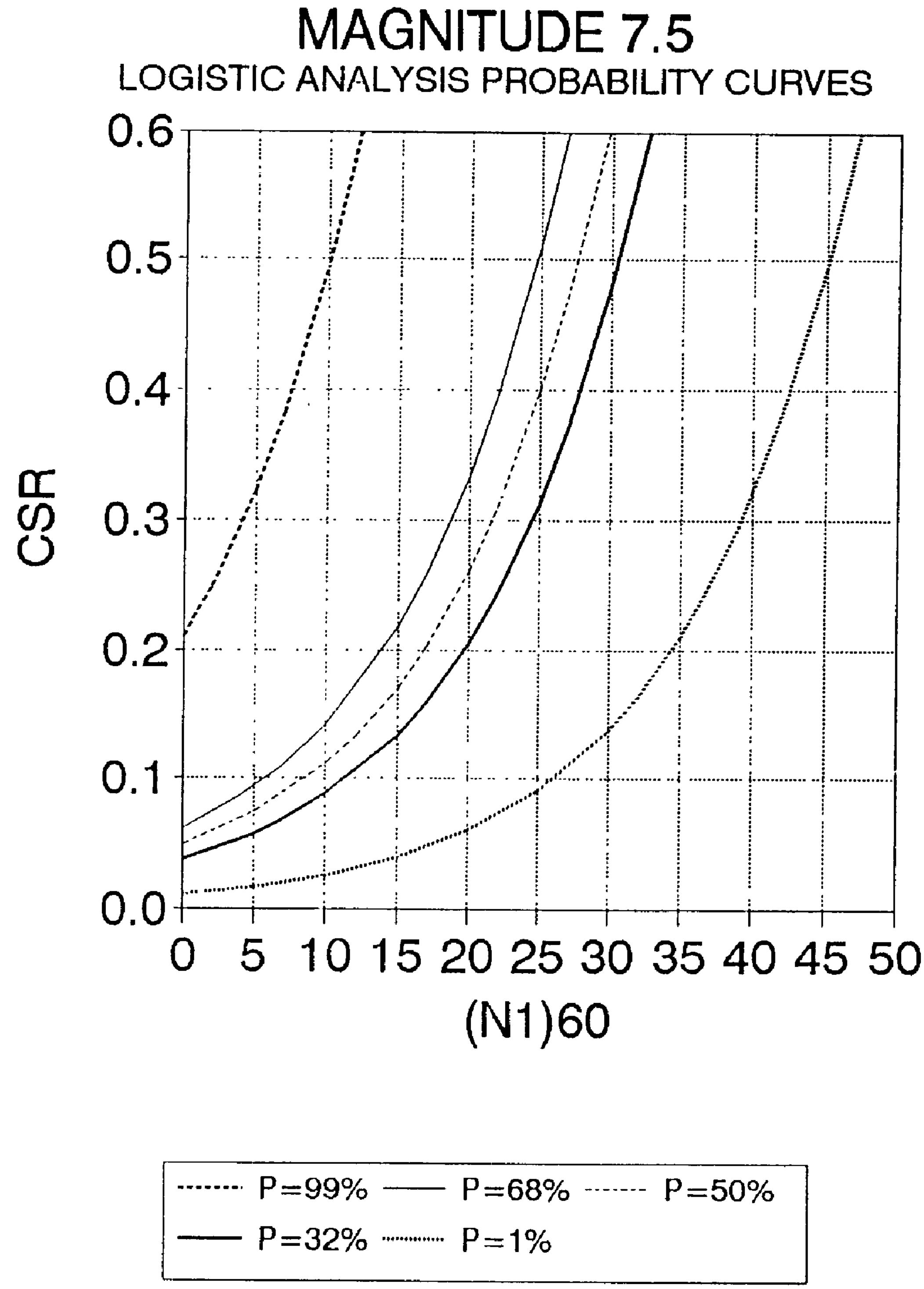

FIGURE 2. Contours of equal probabilities of liquefaction occurrence for M_w=7.5

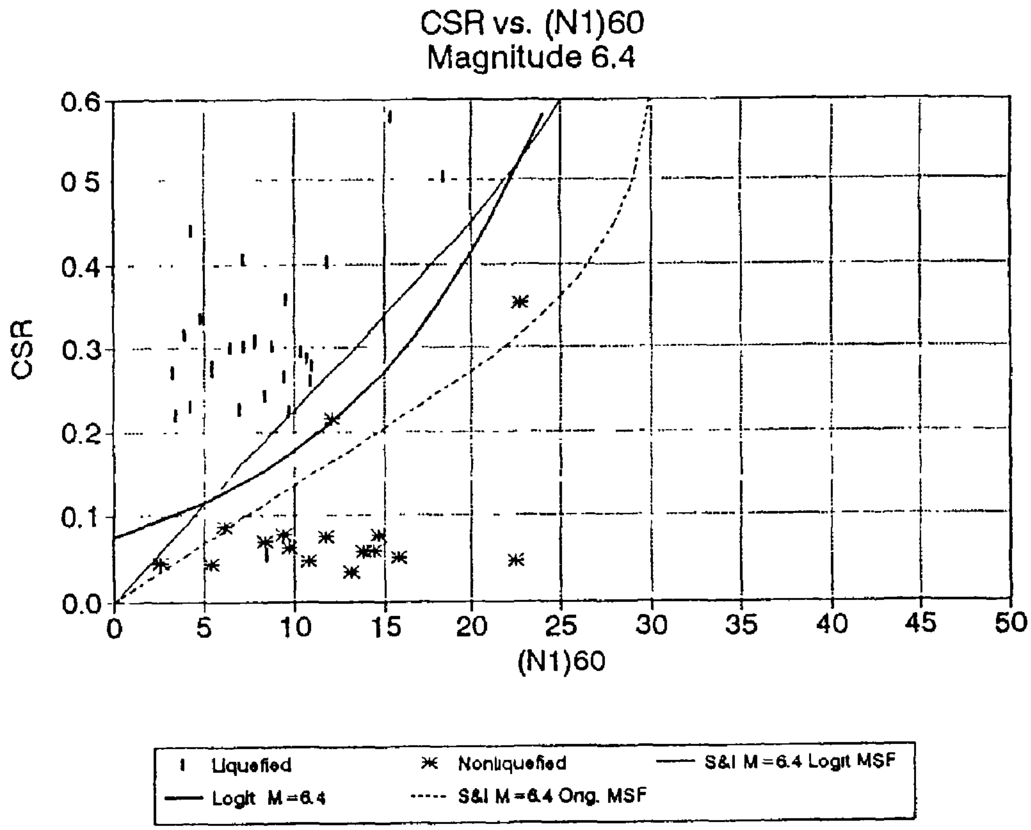

FIGURE 3. A comparison between Logistic curve and Seed and Idriss curve with recommended MSF for $M_w = 6.4$ case sites